AN ANALYTICAL MODEL OF A DEFERRED AND INCREMENTAL UPDATE STRATEGY FOR SECONDARY INDEXES

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1. Introduction

The advantages of using an index for the retrieval of tuples from a relational database is well known. The benefit of having indexes is offset by the cost of maintaining the indexes in the face of updates. For some updates, the cost of maintaining the index may be greater than the cost of updating the desired set of tuples in the relation. In this paper, we describe a strategy for updating secondary indexes which can be classified as deferred and incremental. By deferred, we mean that the index is not updated when the user's update statement is executed. Rather, the appropriate changes are simply recorded for later update of the index. By incremental, we mean that only certain recorded changes will be applied to the index at a given time. We stress that only the index updates are deferred and that the tuple updates are performed by the user's update statement.

2. Previous Research

In [5], the use of a differential file is proposed. The differential file stores all updates, leaving the main file unchanged. Eventually, the differential file and main file are merged. In [5], they were interested in designing a Bloom filter which would indicate whether a given record could be found in the differential file.

In [3,6], the use of a differential file for updating a main file which is organized as a B+ tree is presented. In both [3] and [6], the approach is to perform an efficient batch update of the tree structured main file. In [3], they are concerned with determining the optimal time at which the batch update should take place. In [6], they use a tree structure for the differential file and assume that it can reside in main memory.

Instead of performing a batch update, others [1] have proposed to do incremental updating of the main file. Usually an update will be triggered by a query. In [1], the differential file does not contain the updated records but the update procedure.

In [2], an incremental and deferred update strategy is presented for the maintenance of text indexes. Their emphasis is on concurrency control methods and efficient data structures for the deferred update information.

3. Deferred and Incremental Index Update

Our deferred and incremental index update approach uses a differential file which
grows as updates are executed and shrinks as queries are executed. A record in the differential file consists of the following attributes: a TID, the old-key value and the new-key value. For our purpose, we assume that the differential file is a simple sequential file with an update in-place capability. In addition, a record with a particular TID value occurs only once in the differential file.

The update procedure is simple and is shown below.

1. Access tuples to be modified (via a single index or file scan). If access is through the secondary index, then the query procedure (which follows) must be invoked.
2. Modify tuples.
3. For each indexed attribute, of the modified tuples, that has been modified:
   - if the TID for this tuple does not appear in the differential file
     then write a new record to the differential file
   - else update the new-key value for an existing record in the differential file

A naive query procedure, which uses the secondary index as the access method, is shown below.

1. Follow the appropriate path from the root to a leaf page of the index, (the leaf page contains key values and their associated TID lists).
2. Search the differential file for key values which match the requested key values contained on the leaf page.
   - 2.1 If the old-key value, from a record in the differential file, matches then delete the corresponding TID value from the TID list for the key and set the old-key value to null.
   - 2.2 If the new-key value, from a record in the differential file, matches then
     - if the old-key value is null
       then
       insert the corresponding TID value in the TID list for the new key and set the new-key value to null
     - else
       save the TID value in a temporary TID list.
   - 2.3 If both the new-key value and the old-key value, for a record in the differential file are null, then delete that record from the differential file.
3. If necessary, access the next leaf page and go to step 2.
4. Retrieve the tuples for the TID lists which have been found (including the temporary TID list). The TID lists contain any modifications as performed in step 2.

In [4], we prove that only one record in the differential file is needed per tuple, regardless of the number of key value changes that have taken place on any given tuple. We also prove in [4], that any TID will occur in the index at most one time and possibly be absent from the index during the deferred update.

4. Analytical Model of Differential File Size

Let us denote \( P[\text{size} = s] \) as the probability that the differential file size is \( s \), for
\( s = 0, 1, ..., S_{\text{max}} \) and \( P[i \rightarrow s] \) as the probability that the size of the differential file before the transaction is \( i \), and after the transaction it is \( s \). For a single update, a new record will be added to the differential file if and only if the TID of the record being updated is not contained in any of the existing differential file records. For a search transaction (single key or range of keys), if the old-key value for a differential file record is matched, then this value is set to \( \emptyset \) and the indexed file is modified. A match on the new-key value will result in the record being deleted if and only if the old-key value in that record is \( \emptyset \). Therefore, \( P[i \rightarrow s] \) is determined by the number of records with the old-key value of \( \emptyset \).

To characterize the above phenomenon, we use \( P[\text{empty} \mid s] \) to denote the proportion of records with an empty old-key value when the differential file contains \( s \) records. It is obvious that for each record in the differential file of size \( s \), the probability that its old-key value is empty is also \( P[\text{empty} \mid s] \). The search transaction is treated as if it consisted of two steps. First, when the old-key value is matched, it is set to \( \emptyset \). This results in \( P[\text{empty} \mid s] \) being changed to a new value, \( N(s,j) \), where \( j = 1 \) denotes a single key search and \( j = 2, ..., R_{\text{max}} \) denotes a range search consisting of 2 keys through \( R_{\text{max}} \) keys. The variable \( R_{\text{max}} \) denotes the maximum number of keys in a range query. The calculation of \( N(s,j) \) will be given later. The second step is to match the new-key value. It should be noted that \( P[\text{empty} \mid s] \) changes from transaction to transaction. Another consideration is, that if \( P[\text{size} = s] = 0 \) then \( P[\text{empty} \mid s] \) is undefined. The above two considerations direct us in the calculation of \( P^{(t)}[\text{empty} \mid s] \) where \( t \) denotes that \( t \) transactions have been processed. We use \( P^{(t-1)}[\text{empty} \mid s] \) for all states \( s' \) that can reach \( s \). That is, the differential file can change from a state having \( s' \) records to a state having \( s \) records. Also, \( P^{(t-1)}[\text{empty} \mid s] \) is undefined if \( P^{(t-1)}[\text{size} = s] = 0 \).

Next, we develop an iteration model to calculate \( P[\text{size} = s] \) based on \( P[\text{empty} \mid s] \). Also, the maximum size of the differential file after \( t \) transactions have been processed can be easily predicted from the size distribution. We use the following notation in our model:

- \( \text{Prob[up]} \) = probability of update
- \( \text{Prob[ss]} \) = probability of single key search
- \( \text{Prob[rs]} \) = probability of range search
- \( K_{\text{max}} \) = number of unique key values
- \( S_{\text{max}} \) = maximum size (in records)
- \( R_{\text{max}} \) = maximum number of keys in range search

Initially, we set \( P^{(0)}[\text{size} = 0] = 1 \) and \( P^{(0)}[\text{size} = i] = 0 \) for \( i = 1, ..., S_{\text{max}} \). All \( P^{(0)}[\text{empty} \mid s] \) are undefined for \( s = 1, ..., S_{\text{max}} \). Iterate over equations (1 - 5), i.e., \( t = 1, ..., \text{ceil} \), until the difference between the average size of the differential file in two successive iterations is less than an epsilon (\( \epsilon = 10^{-4} \)) value. The probability that after \( t \) transactions, the differential file size is \( s \), is shown in equation 1.

\[
P^{(t)}[\text{size} = s] = \sum_{i=0}^{S_{\text{max}}} P^{(t-1)}[\text{size} = i] \cdot P^{(t-1)}[i \rightarrow s]
\] (1)
where $P^{(t-1)}[i \rightarrow s]$ is computed by equation 4.

$P^{(t)}[\text{empty} \mid s]$, in equation 2, is defined in a somewhat similar manner, except now, we have to take into account $Q^{(t-1)}[i \rightarrow s]$ for every $i$ value that can lead us to a state with $s$ records. $Q^{(t)}[i \rightarrow s]$, is defined in equation 5.

\[
P^{(t)}[\text{empty} \mid s] = \begin{cases} 
\text{undefined,} & \text{if } P^{(t)}[\text{size}=s] = 0 \\
\frac{s_{\text{max}} \sum_{i=0}^{s_{\text{max}}} P^{(t-1)}[\text{size}=i] \cdot P^{(t-1)}[i \rightarrow s] \cdot Q^{(t-1)}[i \rightarrow s]}{\sum_{j=0}^{s_{\text{max}}} P^{(t-1)}[\text{size}=j] \cdot P^{(t-1)}[j \rightarrow s]} & \text{otherwise}
\end{cases}
\]

(2b)

where $Q^{(t-1)}[i \rightarrow s]$ is defined in equation 5.

The following equation yields the new proportion of records with an old-key value of $\varnothing$. This is relative to the differential file size and the number of keys which appear in a search request. This formula will be used in defining $Q^{(t)}[i \rightarrow s]$ as shown shortly.

\[
N^{(t-1)}(s,j) = P^{(t-1)}[\text{empty} \mid s] + (1 - P^{(t-1)}[\text{empty} \mid s]) \cdot \frac{i}{K_{\text{max}}}
\]

(3)

for $s=1,\ldots,s_{\text{max}}$ such that $P^{(t-1)}[\text{size}=s] > 0$ & $j=1,\ldots,R_{\text{max}}$.

Equations 4(a - d) depict the 4 possibilities for a transition in the differential file size.

\[
P^{(t-1)}[s \rightarrow s+1] = \text{Prob}[\text{up}] \cdot (1 - \frac{s}{s_{\text{max}}}) \quad \text{(if } P^{(t-1)}[\text{size}=s] > 0) \]

(4a)

\[
P^{(t-1)}[s \rightarrow s] = \text{Prob}[\text{up}] \cdot \frac{s}{s_{\text{max}}} + \text{Prob}[\text{ss}] \left[ 1 - N^{(t-1)}(s,1) \cdot \frac{1}{K_{\text{max}}} \right]^s \]

\[
+ \text{Prob}[\text{rs}] \cdot \sum_{j=2}^{R_{\text{max}}-1} \frac{1}{R_{\text{max}}-1} \left[ 1 - N^{(t-1)}(s,j) \cdot \frac{j}{K_{\text{max}}} \right]^s \quad \text{(if } P^{(t-1)}[\text{size}=s] > 0) \]

(4b)

\[
P^{(t-1)}[s \rightarrow s-\delta] = \text{Prob}[\text{ss}] \cdot \left[ \frac{s}{\delta} \right] N^{(t-1)}(s,1) \cdot \frac{1}{K_{\text{max}}} \left[ 1 - N^{(t-1)}(s,1) \cdot \frac{1}{K_{\text{max}}} \right]^{s-\delta}
\]

\[
+ \text{Prob}[\text{rs}] \cdot \sum_{j=2}^{R_{\text{max}}-1} \frac{1}{R_{\text{max}}-1} \left[ \frac{s}{\delta} \right] N^{(t-1)}(s,j) \cdot \frac{j}{K_{\text{max}}} \left[ 1 - N^{(t-1)}(s,j) \cdot \frac{j}{K_{\text{max}}} \right]^{s-\delta} \quad \text{(if } P^{(t-1)}[\text{size}=s] > 0 \text{ & for } \delta=1,\ldots,s) \]

(4c)

\[
P^{(t-1)}[i \rightarrow j] = 0 \quad \text{for all other cases}
\]

(4d)

Again, we consider the possible state transitions for the differential file in equations 5(a - e). Except, that here we calculate the proportion of records in the
differential file whose old-key value is \( \emptyset \).

\[ Q^{(t-1)}[0 \rightarrow 0] \text{ is not defined and is never used} \quad (5a) \]

\[ Q^{(t-1)}[0 \rightarrow 1] = 0 \quad (5b) \]

\[ Q^{(t-1)}[s \rightarrow s] = Prob[up] \cdot P^{(t-1)}[\text{empty } | s] + Prob[ss] \cdot N^{(t-1)}(s, 1) \quad (5c) \]

\[ + \frac{\text{ Prob}[rs]}{\text{ Prob}[ss] + \text{ Prob}[rs]} \sum_{j=2}^{R} \frac{1}{R_{\text{max}}-1} N^{(t-1)}(s, j) \quad (\text{if } P^{(t-1)}[\text{size } = s] > 0) \]

\[ Q^{(t-1)}[s \rightarrow s - \delta] = \frac{\text{ Prob}[ss]}{\text{ Prob}[ss] + \text{ Prob}[rs]} \frac{s \cdot N^{(t-1)}(s, 1) - \delta}{s - \delta} \quad (5d) \]

\[ + \frac{\text{ Prob}[rs]}{\text{ Prob}[ss] + \text{ Prob}[rs]} \sum_{j=2}^{R} \frac{1}{R_{\text{max}}-1} \frac{s \cdot N^{(t-1)}(s, j) - \delta}{s - \delta} \quad (\text{if } P^{(t-1)}[\text{size } = s] > 0) \]

\[ Q^{(t-1)}[s \rightarrow s'] = 0 \quad \text{for all other cases.} \quad (5e) \]

In [4], we compare our analytical model with a simulation model. We show that the expected maximum differential file size (in number of records) is only 16% of the number of records in the relation. In addition, we show in [4] that the model differs from the simulation by approximately 15% for the maximum differential file size.

References


\[ 0 = \lim_{\lambda \to 0} \left[ \frac{f(x, \lambda)}{d} \right] \]

\[ \frac{d}{dx} \left[ f(x, \lambda) \right] = \frac{1}{\lambda} \left[ \sum_{k=1}^{n} \frac{x_k}{2} \right] \]

\[ 0 < x = \text{param}^{(1-\lambda)} \]

\[ \frac{d}{dx} \left[ f(x, \lambda) \right] = \frac{1}{\lambda} \left[ \sum_{k=1}^{n} \frac{x_k}{2} \right] \]

\[ 0 = \lim_{\lambda \to 0} \left[ \frac{f(x, \lambda)}{d} \right] \]

For all other cases,

\[ 0 = \lim_{\lambda \to 0} \left[ \frac{f(x, \lambda)}{d} \right] \]

**References**


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